Mechanical Vibration
ارتعاشات مکانیکی(درس دهم)

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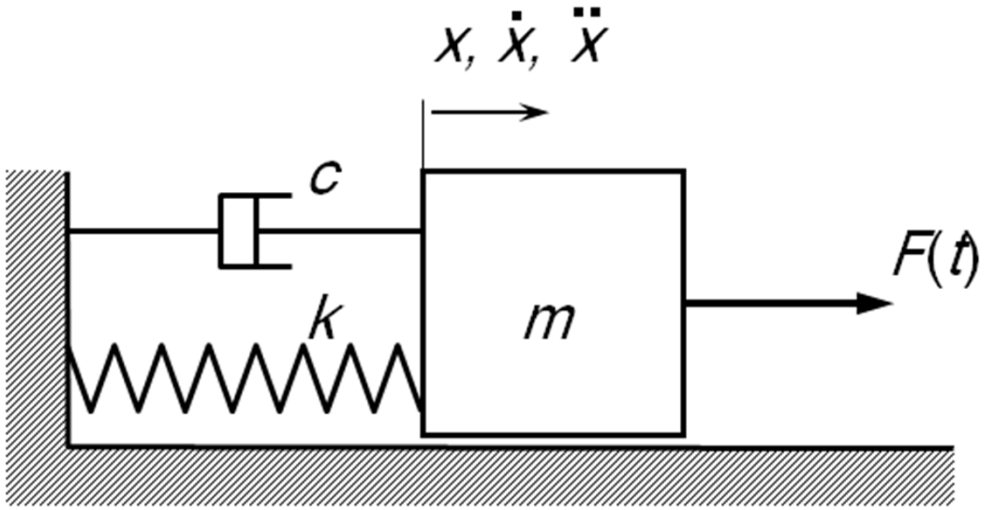
Isfahan University of Technology

ارتعاش اجباری سیستمهای یک درجه آزادی

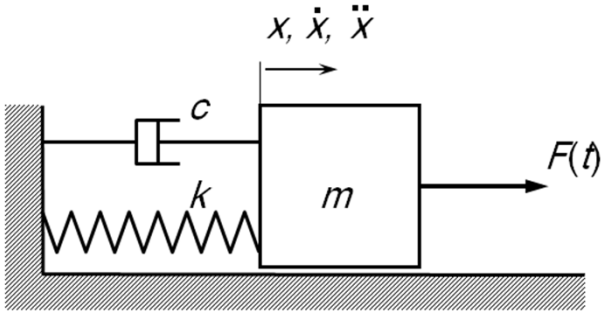


معادله دیفرانسیل ناهمگن $m\ddot{x} + c\dot{x} + kx = F(t)$

$x_h(t)$ حل همگن

$x_p(t)$ حل خصوصی

ارتعاش اجباری سیستمهای یک درجه آزادی



$$m\ddot{x} + c\dot{x} + kx = F_0 Cos(\omega t)$$

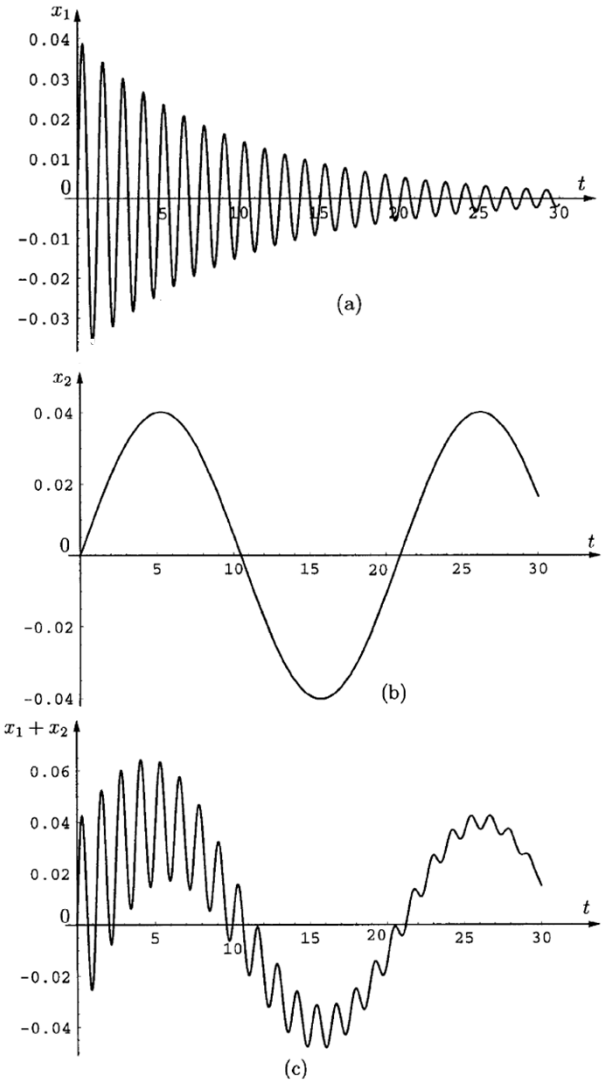
$x_h(t)$

حل همگن

$x_p(t)$

حل خصوصی

$$x(t) = x_h(t) + x_p(t$$



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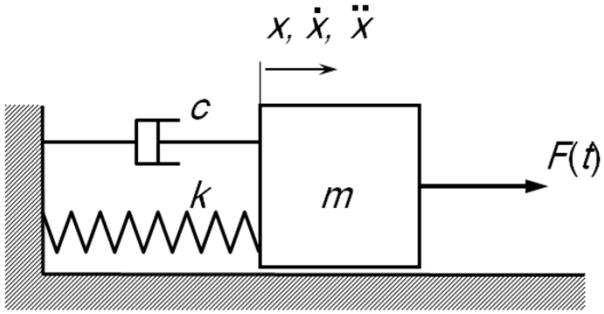

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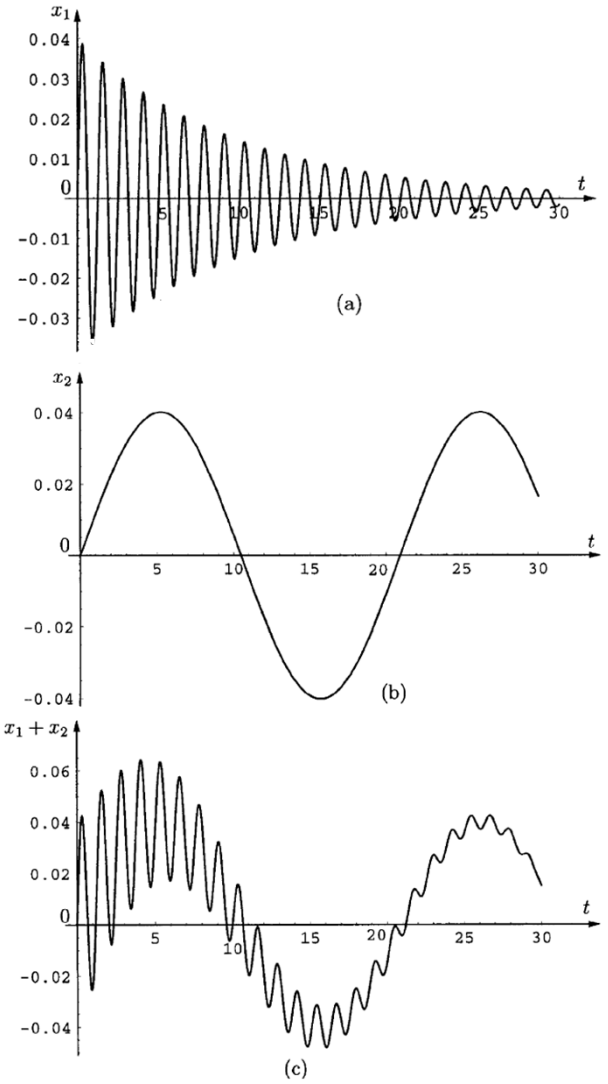
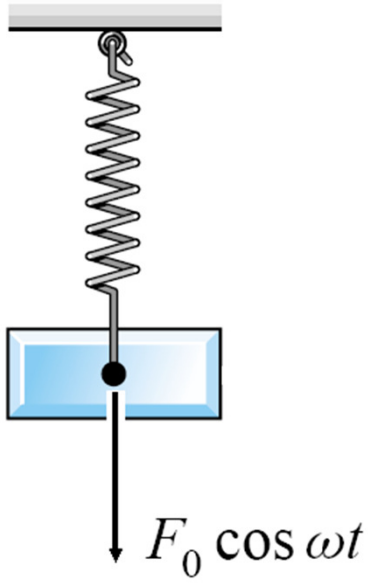


ارتعاش اجباری سیستمهای یک درجه آزادی بدون استهلاک



$$m\ddot{x} + c\dot{x} + kx = F_0 Cos(\omega t)$$ no damping

$$m\ddot{x} + kx = F_0 Cos(\omega t)$$

Case 1 $\omega_0 = \sqrt{k/m} \neq \omega$

$$x = A_1 \sin(\omega_n t) + A_2 \cos(\omega_n t) + x_p$$

$$x_p = XCos(\omega t)$$

$$\ddot{x}_p = -X\omega^2 Cos(\omega t)$$

$$X(\omega_0^2 - \omega^2) = \frac{F_0}{m} \quad \Rightarrow \quad X = \frac{\frac{F_0}{m}}{\omega_0^2 - \omega^2}$$

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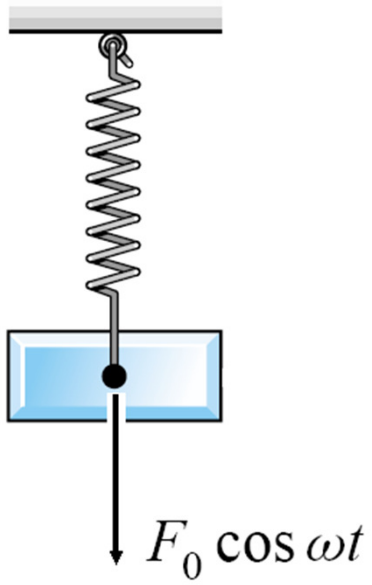


$$m\ddot{x} + kx = F_0 Cos(\omega t)$$

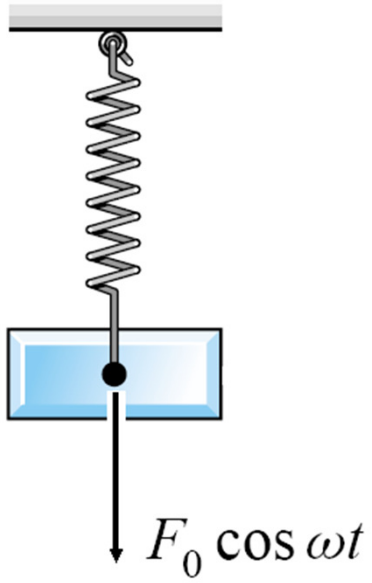
Case 1 $\omega_0 = \sqrt{k/m} \neq \omega$

$$x = A_1 \sin(\omega_n t) + A_2 \cos(\omega_n t) + \frac{f_0}{\omega_n^2 - \omega^2} Cos(\omega t)$$

$$f_0 = F_0 / m$$

$$t = 0 \Rightarrow \begin{cases} x = x_0 \\ \dot{x} = v_0 \end{cases}$$

ارتعاش اجباری سیستمهای یک درجه آزادی بدون استهلاک



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$$f_0 = F_0 / m$$

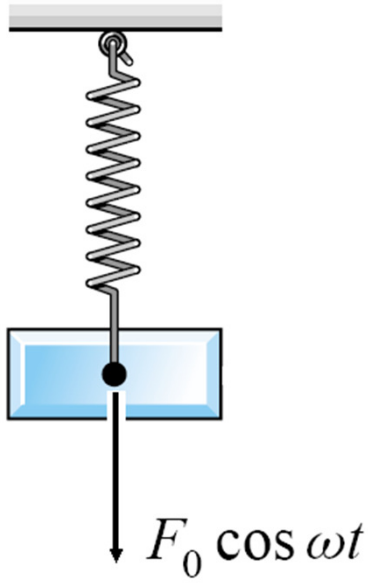
$$t = 0 \Rightarrow \begin{cases} x = x_0 \\ \dot{x} = v_0 \end{cases}$$

$$x(0) = A_2 + \frac{f_0}{\omega_n^2 - \omega^2} = x_0$$

$$\dot{x}(0) = \omega_n A_1 = v_0$$

$$x(t) = \frac{v_0}{\omega_n} \sin \omega_n t + \left(x_0 - \frac{f_0}{\omega_n^2 - \omega^2} \right) \cos \omega_n t + \frac{f_0}{\omega_n^2 - \omega^2} \cos \omega t$$

ارتعاش اجباری سیستمهای یک درجه آزادی بدون استهلاک



$$m\ddot{x} + kx = F_0 Cos(\omega t)$$

$$x(t) = \frac{v_0}{\omega_n} \sin \omega_n t + \left(x_0 - \frac{f_0}{\omega_n^2 - \omega^2} \right) \cos \omega_n t + \frac{f_0}{\omega_n^2 - \omega^2} \cos \omega t$$

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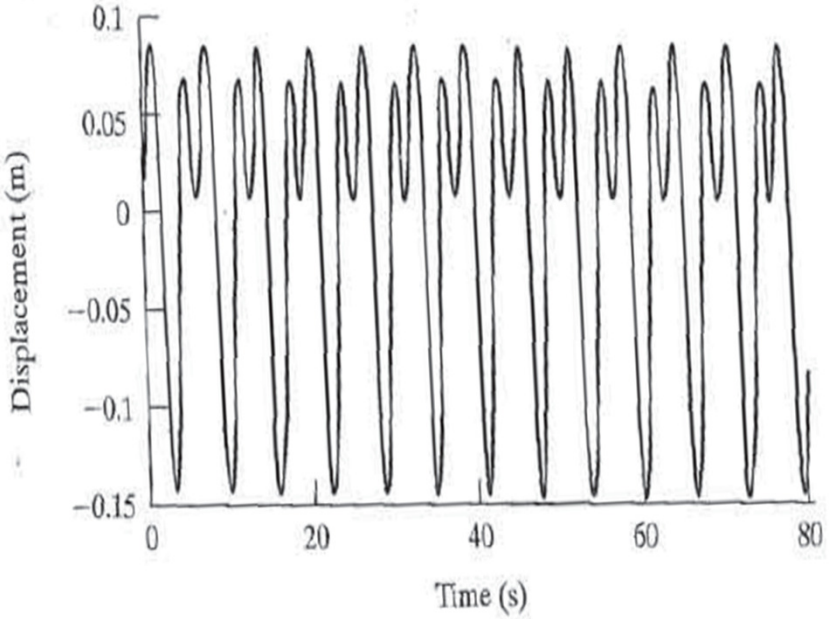
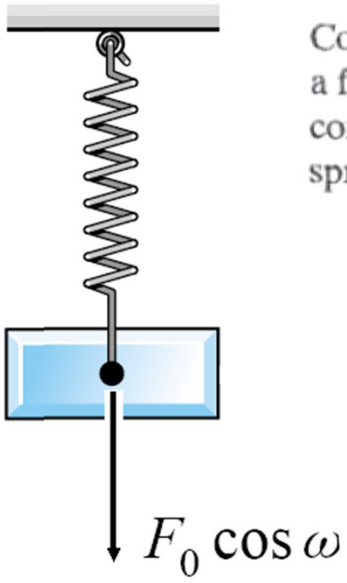


ارتعاش اجباری سیستمهای یک درجه آزادی بدون استهلاک

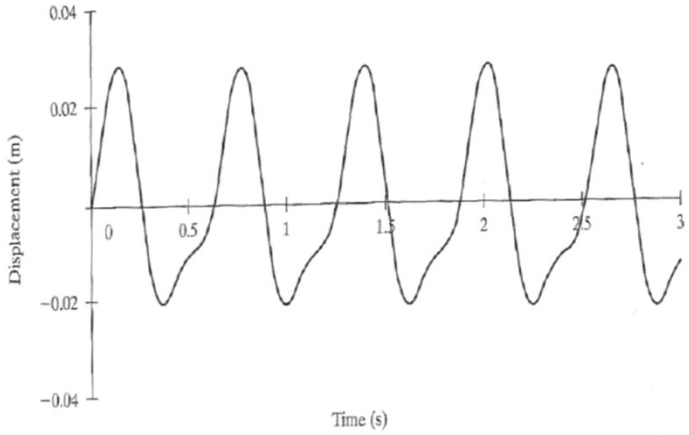


Compute and plot the response of a spring–mass system modeled by equation (2.2) to a force of magnitude 23 N, driving frequency of twice the natural frequency, and initial conditions given by $x_0 = 0$ m and $v_0 = 0.2$ m/s. The mass of the system is 10 kg and the spring stiffness is 1000 N/m.

$$\omega_n = \sqrt{\frac{1000\ \text{N/m}}{10\ \text{kg}}} = 10\ \text{rad/s},\ \omega = 2\omega_n = 20\ \text{rad/s},\ f_0 = \frac{23\ \text{N}}{10\ \text{kg}} = 2.3\ \text{N/kg}$$

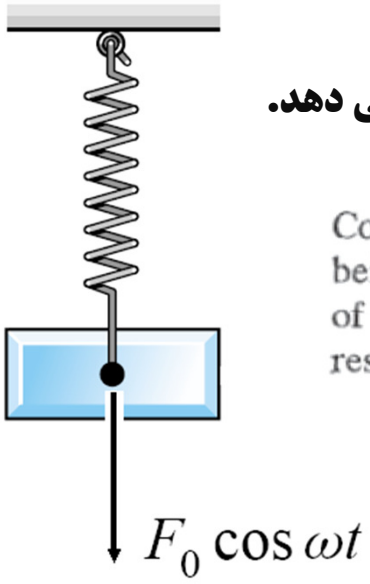
$$\frac{v_0}{\omega_n} = \frac{0.2\ \text{m/s}}{10\ \text{rad/s}} = 0.02\ \text{m},\ \frac{f_0}{\omega_n^2 - \omega^2} = \frac{2.3\ \text{N/kg}}{(10^2 - 20^2)\text{rad}^2/\text{s}^2} = -7.9667 \times 10^{-3}\ \text{m}$$

$$x(t) = 0.02 \sin 10t + 7.667 \times 10^{-3}(\cos 10t - \cos 20t)\ \text{m}$$



ارتعاش اجباری سیستمهای یک درجه آزادی بدون استهلاک

پدیده ضربان:
در صورتی که فرکانس نیروی خارجی نزدیک فرکانس طبیعی سیستم باشد، این پدیده رخ می دهد.



Consider the forced vibration of a mass m connected to a spring of stiffness 2000 N/m being driven by a 20-N harmonic force at 10 Hz (20π rad/s). The maximum amplitude of vibration is measured to be 0.1 m and the motion is assumed to have started from rest ($x_0 = v_0 = 0$). Calculate the mass of the system.

$$x(t) = \frac{f_0}{\omega_n^2 - \omega^2}(\cos \omega t - \cos \omega_n t)$$

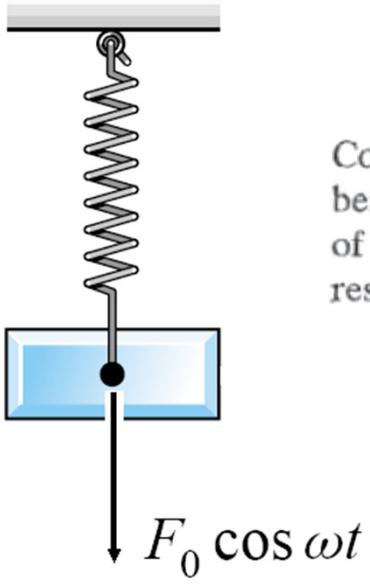
$$x(t) = \frac{2f_0}{\omega_n^2 - \omega^2} \sin\left(\frac{\omega_n - \omega}{2} t\right) \sin\left(\frac{\omega_n + \omega}{2} t\right)$$

$$\frac{2f_0}{\omega_n^2 - \omega^2} = 0.1 \text{ m}$$

$$m = \frac{(0.1 \text{ m})(2000 \text{ N/m}) - 2(20 \text{ N})}{(0.1 \text{ m})(10 \times 2\pi \text{ rad/s})^2} = \frac{4}{\pi^2} = 0.405 \text{ kg}$$

ارتعاش اجباری سیستمهای یک درجه آزادی بدون استهلاک

پدیده ضربان:



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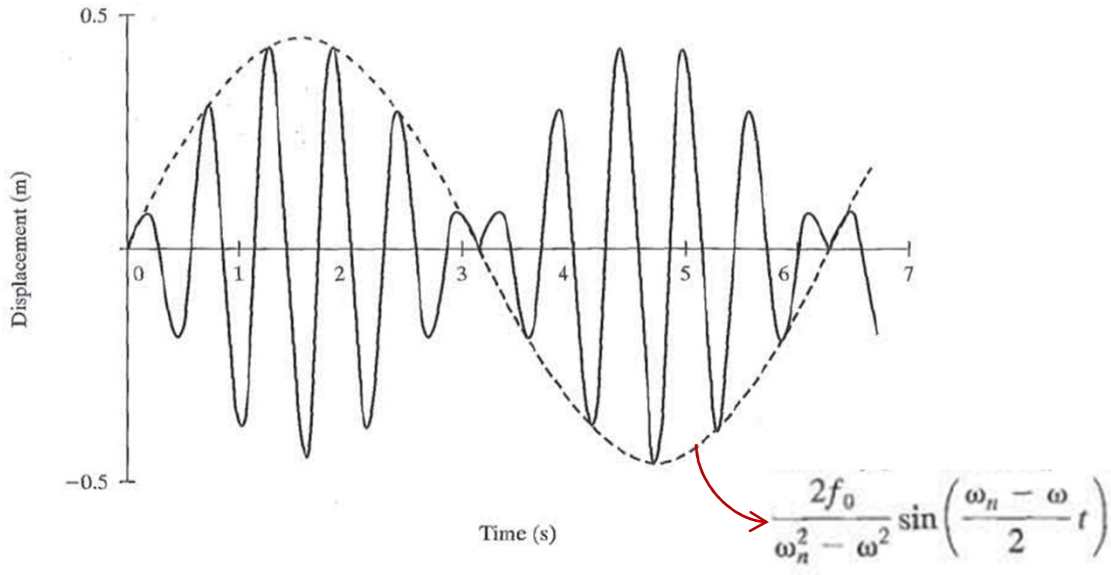
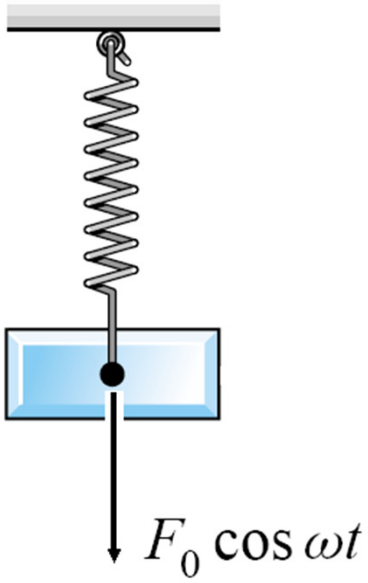


ارتعاش اجباری سیستمهای یک درجه آزادی بدون استهلاک

پدیده تشدید:



Case 2 $\omega = \omega_n$

$$m\ddot{x} + kx = F_0 Cos(\omega_n t)$$

$$x_p(t) = tX \sin \omega t$$

$$x_p(t) = \frac{f_0}{2\omega} t \sin \omega t$$

$$x(t) = A_1 \sin \omega t + A_2 \cos \omega t + \frac{f_0}{2\omega} t \sin \omega t$$

$$x(t) = \frac{v_0}{\omega} \sin \omega t + x_0 \cos \omega t + \frac{f_0}{2\omega} t \sin \omega t$$

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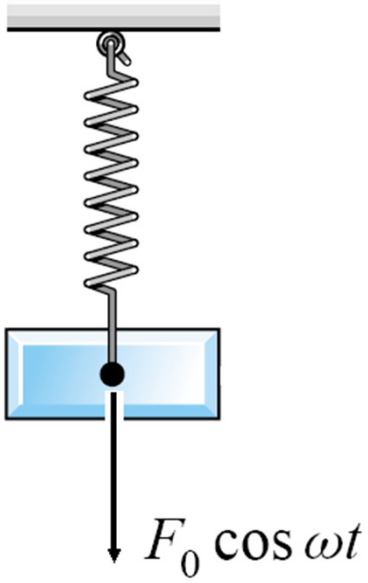

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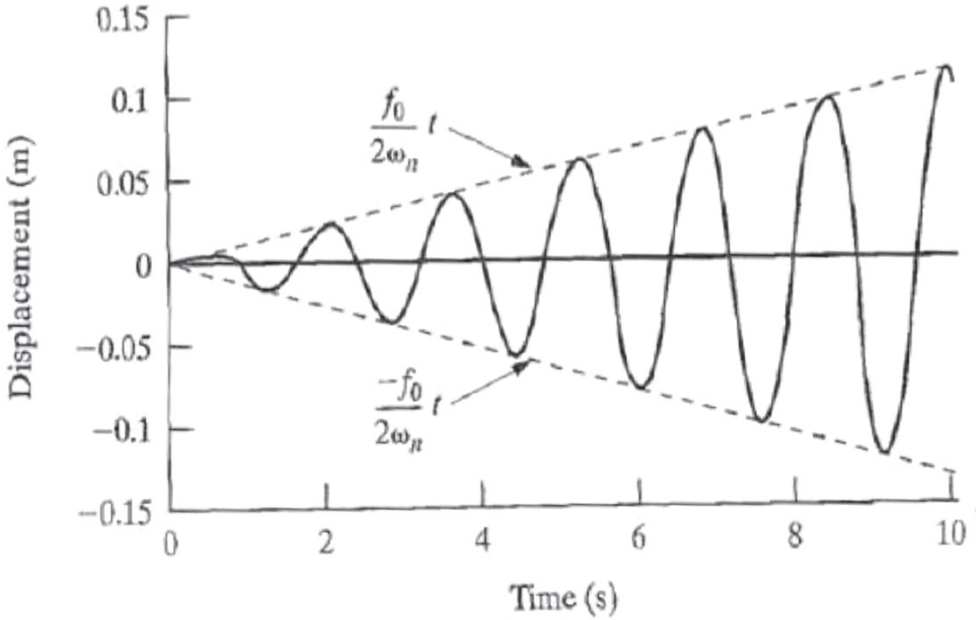

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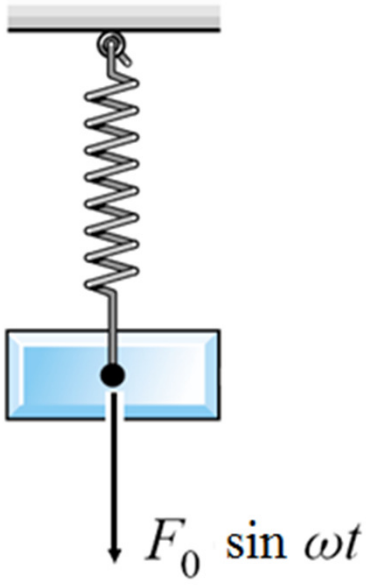

$$m\ddot{x}(t) + kx(t) = F_0 \sin \omega t \quad \text{or} \quad \ddot{x}(t) + \omega_n^2 x(t) = f_0 \sin \omega t$$

$$x_p(t) = X \sin \omega t \qquad -\omega^2 X \sin \omega t + \omega_n^2 X \sin \omega t = f_0 \sin \omega t$$

$$X = \frac{f_0}{\omega_n^2 - \omega^2}$$

$$x_p(t) = \frac{f_0}{\omega_n^2 - \omega^2} \sin \omega t$$

$$x(t) = A_1 \sin \omega_n t + A_2 \cos \omega_n t + \frac{f_0}{\omega_n^2 - \omega^2} \sin \omega t$$

$$x(0) = x_0 = A_2 \quad \text{and} \quad \dot{x}(t) = \omega_n A_1 + \frac{\omega f_0}{\omega_n^2 - \omega^2} = v_0$$

$$\Rightarrow A_1 = \frac{v_0}{\omega_n} - \frac{\omega}{\omega_n} \frac{f_0}{\omega_n^2 - \omega^2} \quad \text{and} \quad A_2 = x_0$$

$$x(t) = x_0 \cos \omega_n t + \left(\frac{v_0}{\omega_n} - \frac{\omega}{\omega_n} \frac{f_0}{\omega_n^2 - \omega^2} \right) \sin \omega_n t + \frac{f_0}{\omega_n^2 - \omega^2} \sin \omega t$$